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Simplified Approach for Control of Rotating Stall Part 1: Theoretical Development

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In this article the theoretical foundations of a simplified approach for control of rotating stall are presented. This approach requires two-dimensional sensing, but only a single one-dimensional axisymmetric effector with relatively low bandwidth requirements. The reduced actuation requirements of this approach are a consequence of the fact that in this approach one does not require or act upon rotating stall phase information. This is due to the fact that one does not seek to extend the theoretical stable axisymmetric flow range of the compressor. Rather, one seeks to directly address persistent disturbances that would otherwise throttle the equilibrium into the unstable axisymmetric flow range of the compressor. In addition, one seeks to enlarge the domains of attraction of linearly stable axisymmetric equilibria, thereby addressing impulsive disturbances that would otherwise perturb the system state beyond the domain of attraction of the stable axisymmetric equilibrium. Experimental validation of this approach on a single-stage low-speed axial compressor rig is discussed in Part 2 of this article.

I. Introduction

THE compressor aerodynamic instabilities of rotating stall and surge correspond to relatively high-frequency and high-amplitude flow oscillations.[1–4] Rotating stall is an inherently two-dimensional localized compression system oscillation that involves a circumferentially rotating partial annular flow blockage. Surge is an essentially one-dimensional global compression system oscillation that involves axial flow oscillations and in some cases even axial flow reversals. At the very least, these oscillations result in a dramatic loss of compression system performance and operating efficiency. However, in some instances these oscillations can result in catastrophic system failure. For these reasons, rotating stall and surge must be avoided at all costs in compressor operation.

Compressor stall margin is used to accommodate compression system uncertainties and disturbances that would otherwise cause the system to enter an unstable operating regime resulting in rotating stall or surge. These uncertainties include modeling errors, in-service deterioration and manufacturing quality variations, to name a few. The uncertainties can be characterized as either unmodeled dynamics, parameter ignorance, or slowly time-varying parameters. Compression system disturbances include transients, inlet distortion, customer bleed, and internal noise (such as combustion noise), etc. These disturbances can be characterized as either impulsive disturbances or persistent disturbances. Impulsive disturbances perturb the system state momentarily from the current equilibrium without throttling the system equilibrium. Persistent disturbances throttle the system equilibrium, thereby creating a new system equilibrium (or set of equilibria). To accommodate these uncertainties/disturbances the stall margin restricts demanded compressor operation to a region of stable axisymmetric compressor operation, at a "safe" margin from the stall line,[5] which represents the boundary between the regions of locally stable and unstable axisymmetric compressor operation. The use of a stall margin to ensure safe compressor operation introduces a loss in performance and design opportunity. Reduction or elimination of the compressor stall margin through the use of an active stall controller[6–22] is the major goal of a more modern approach for addressing compressor stall phenomena.

The previous discussion on active stall control may give the reader the impression that the most likely use of an active stall controller would be to reduce the operational stall margin for an existing compressor design. While this may be a desirable goal for some compressor designs, in most cases this would be detrimental relative to the efficiency of the compressor. This can be readily seen by examining the existing compressor design process. In conventional compressor design, a design point pressure ratio (based on cycle requirements) and mass flow rate (based on thrust requirements) are specified. A stall margin requirement is also specified. The compressor is then designed to maximize the design point efficiency subject to the stall margin specification. Remaining design freedom is used to address secondary design considerations such as off design performance, weight, cost, etc. Now, if the operational stall margin is reduced through the use of active control, the efficiency of the compressor may, in fact, be reduced. For this reason, we would rather think in terms of using active stall control as part of a control-configured compressor design process, wherein the design process would proceed as follows: 1) specify a design point (pressure ratio and mass flow rate), 2) specify a reduced stall margin requirement, 3) maximize design point efficiency subject to reduced stall margin requirement, 4) use remaining design freedom to address secondary design considerations, and 5) design an active compressor control system to permit safe compressor operation.

This article is concerned with the subject of active stall control. However, in order to keep the scope of this article reasonable, active control of surge is not addressed, and rather only control of rotating stall is addressed. In addition, con-

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sistent with previous work on rotating stall control, we will not address directly the effects of compression system uncertainties in our discussion here on active control of rotating stall. While this is an important consideration, we feel it most important to first address the issues of compression system disturbances. Once these issues are resolved, the issues of compression system uncertainties can be resolved through the robustification of active stall controllers by the application of well-established robust control design methodologies.[23] The approach to rotating stall control developed in this article is distinguished from previous approaches from both a conceptual standpoint and from a hardware standpoint. The key conceptual distinction is that in this approach, compression system disturbances are directly addressed. Both persistent and impulsive disturbances that act on the system are directly addressed in this approach, as opposed to the passive manner in which they are addressed in previous approaches. From a hardware standpoint, the key distinction is that in this approach, only a single one-dimensional axisymmetric effector (which acts on annulus averaged flow properties), is required, as opposed to the two-dimensional nonaxisymmetric actuation (that act on circumferentially resolved flow properties) required in previous approaches. To explain the validity and justification of these two features of the control approach presented in this article, it is necessary to use terminologies and concepts from nonlinear stability theory such as domains of attraction, etc.[24] It is also necessary to be familiar with the disturbance rejection capability of feedback, in addition to its stability augmentation capability.[25]

Previous approaches to rotating stall control[7,6,13,15–17] are based entirely on the notion of linear feedback stabilization, wherein the goal is to locally stabilize unstable axisymmetric equilibrium points, thereby extending the theoretical stable axisymmetric flow range of the compressor. The motivation for this approach is that if a persistent disturbance throttles the equilibrium, the new axisymmetric equilibrium will be stable as long as the equilibrium is not throttled beyond the stabilized flow range. This approach clearly accounts for persistent disturbances. However, it does not fully account for impulsive disturbances because the linear perspective does not address how large a perturbation from a stable equilibrium the system can tolerate and still return to the equilibrium. Thus, there is a need at a given stabilized equilibrium point to assess from a nonlinear perspective how stable the equilibrium point is. This involves determining the domain of attraction of the equilibrium point. Now, if sufficiently large domains of attraction are achieved about the linearly stabilized equilibrium points (e.g., if the points are made global attractors), then this type of controller would be expected to address adequately compression system disturbances in practice (provided the controller is robust enough to accommodate compression system uncertainties). In any case, there is a cost in complexity associated with this control approach with respect to the number of required actuators and sensors and the corresponding bandwidth requirements. Indeed, the linearized dynamics about the unstable axisymmetric equilibrium points are linearly uncontrollable and unobservable from axisymmetric one-dimensional actuation and annulus-averaged sensing, respectively. Thus, nonaxisymmetric two-dimensional actuation and two-dimensional sensing are required for linear feedback stabilization. This in turn implies that multiple actuators and sensors are required for linear feedback stabilization of an unstable axisymmetric equilibrium point. Furthermore, since linear stabilization involves tracking the rotating disturbance to damp it out, two important sensing and actuation issues arise. First, a relatively large number of sensors and actuators are required in order to satisfy the sampling theorem[26] applied spatially relative to the relevant harmonics of the rotating disturbance, so that they can be detected and actuated. Second, the actuator bandwidth requirements are relatively high, since they are dictated by the speed and harmonic content of the rotating disturbance. Moreover, the further the stable axisymmetric flow range is extended, the larger the number of required sensors and actuators and the higher the corresponding bandwidth requirements. This is because higher-order harmonics of rotating stall must be stabilized in order to achieve further extension of the stable axisymmetric flow range.[27] For this reason, it seems worthwhile to seek out an approach for eliminating, or at least reducing, the required extension of the stable axisymmetric flow range.

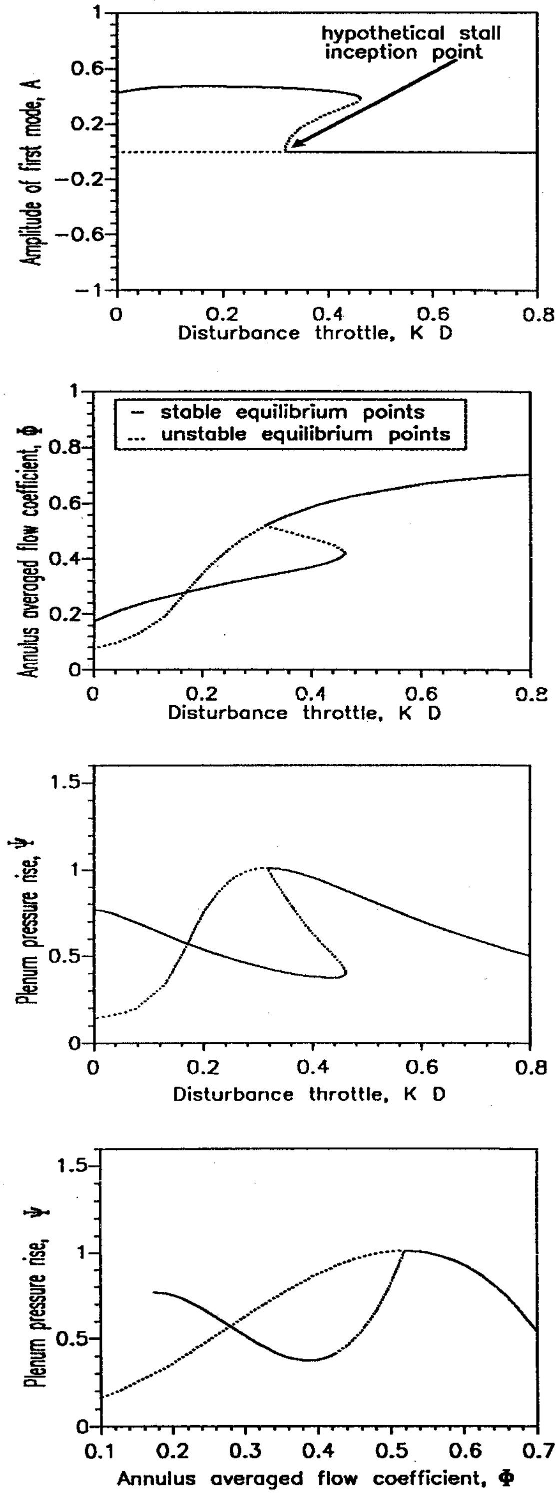

Fig. 1 Uncontrolled system bifurcation diagram.

The simplified approach to control of rotating stall presented in this article is complementary to previous approaches

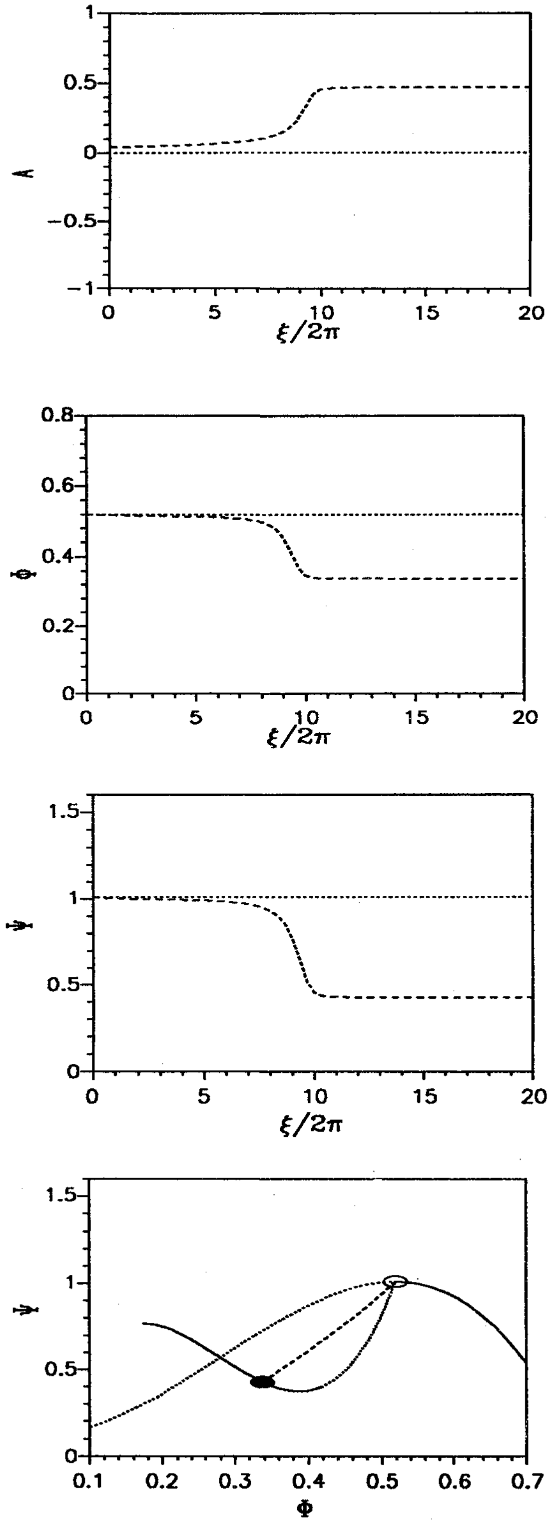

Fig. 2 Uncontrolled system response to nonequilibrium initial condition on A at stall inception point.

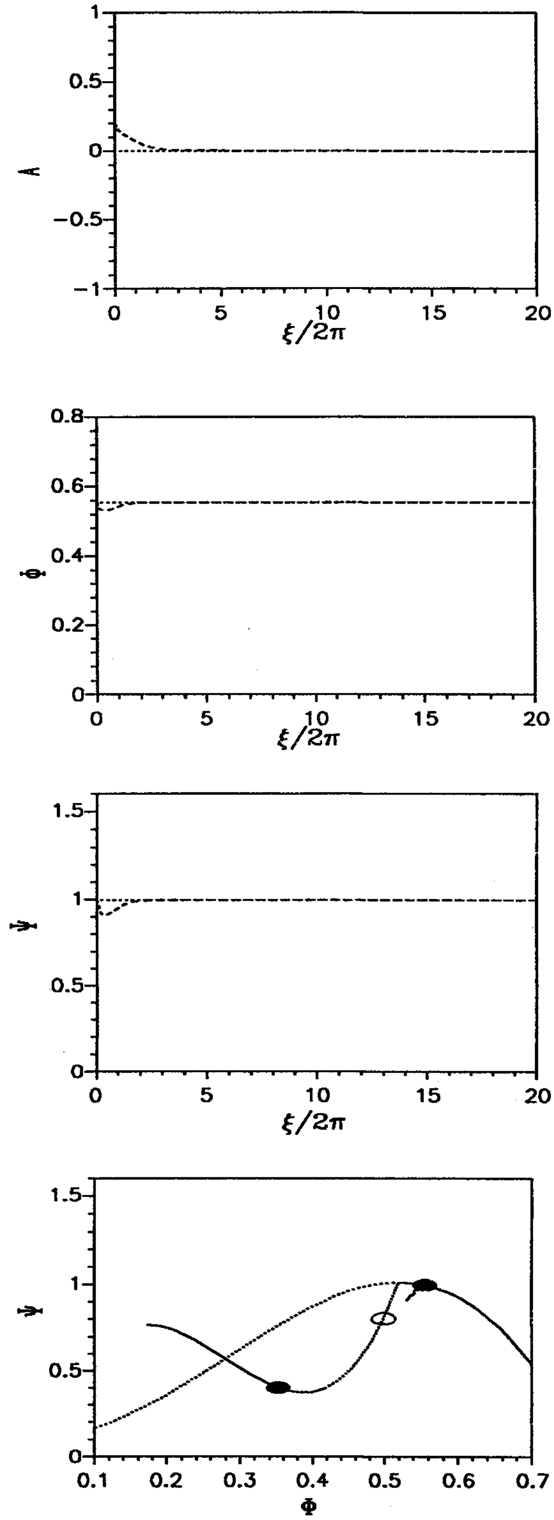

Fig. 3 Uncontrolled system response to small nonequilibrium initial condition on A in hysteresis region.

in that it focuses on reducing, and possibly even eliminating, the required extension of the stable flow range. This is accomplished by directly addressing persistent disturbances. Nonlinear feedback disturbance rejection is used to attenuate the effects of persistent disturbances by reducing the amount that these disturbances throttle the equilibrium point. Thus, a reduction in the required amount of stable axisymmetric flow range extension is achieved. Since the controller does not seek to extend the theoretical stable axisymmetric flow range of the compressor, two-dimensional sensing along with only a single one-dimensional axisymmetric effector are required. Also, the actuator bandwidth requirements are relatively low, since the controller does not require or act upon rotating stall phase information. Impulsive disturbances are also directly addressed in this approach. Nonlinear feedback stability augmentation is used to enlarge domains of attraction about linearly stable axisymmetric equilibria of the uncontrolled compressor. This has the effect of eliminating 1) the abrupt jump in steady-state rotating stall amplitude when the system enters rotating stall and 2) hysteresis with respect to

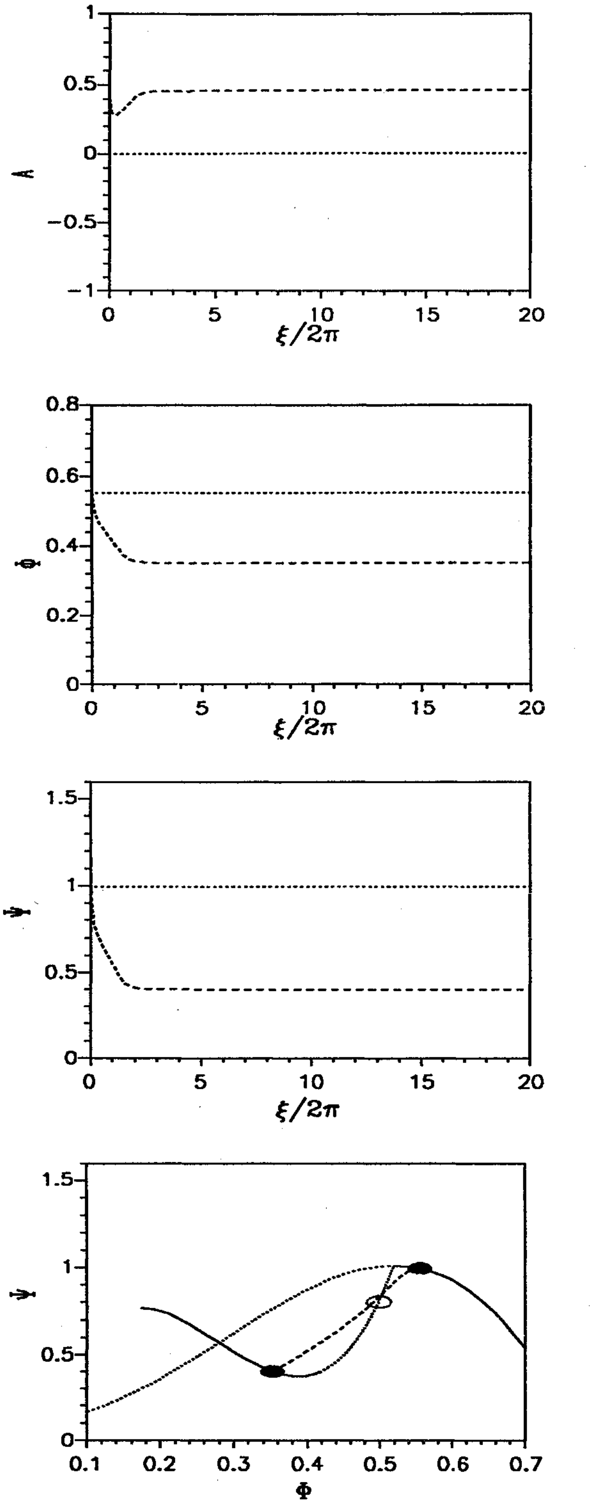

Fig. 4 Uncontrolled system response to large nonequilibrium initial condition on A in hysteresis region.

the onset and cessation of rotating stall. Moreover, by enlarging the domains of attraction of locally stable axisymmetric equilibrium points and making them globally stable, the possibility for impulsive disturbances to perturb the system state beyond the domains of attraction of these points is eliminated, subject to availability of required control authority, and extension of the effective stable axisymmetric flow range is possible. Experimental validation of this approach on a single-stage low-speed axial compressor rig is discussed in the companion article.[28] In addition, a video demonstration of this approach can be found in Refs. 21 and 22.

An outline of this article is given as follows: First, rotating stall is discussed from a global nonlinear perspective to motivate the control approach presented in this article. Second, the simplified approach for rotating stall control presented in this article is contrasted with previous approaches for control of rotating stall.

Throughout this article, key concepts are discussed and key points are made via reference to what we believe to be generic compression system bifurcation diagrams. The specific bifurcation diagrams shown in this article are generated using the first-term Galerkin approximation of the Moore–Greitzer model,[29] which is reviewed in the Appendix. The first-term Galerkin approximation of the Moore–Greitzer model is considered since 1) it involves only three states, thereby allowing the discussion of the phenomena to be kept simple and 2) it provides the correct qualitative description of the phenomena of rotating stall from a nonlinear perspective. Indeed, we believe that the bifurcation diagrams generated with this model are generic and that they are representative of many axial compression systems. In addition, these bifurcation diagrams are not rooted in the model used to generate them, and other models such as a parallel compressor model could also be used to generate them. The maps and parameters used in the model to generate these bifurcation diagrams correspond to

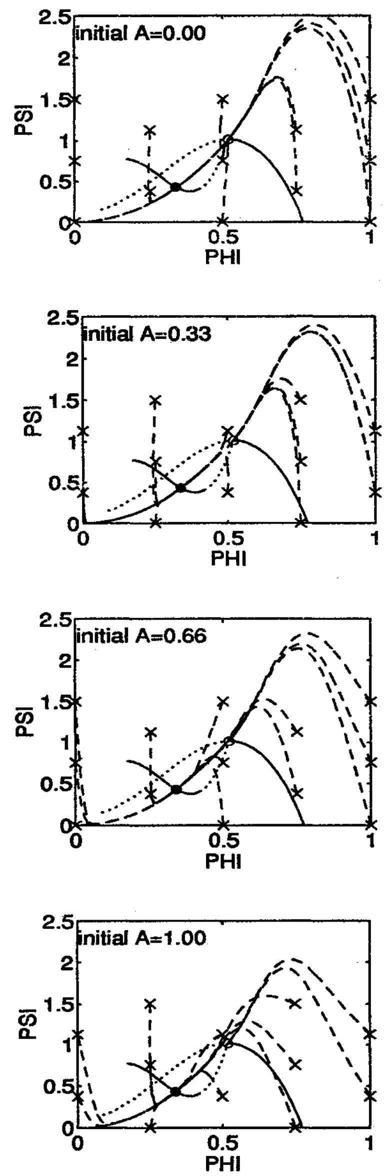

Fig. 5 Uncontrolled system trajectories (throttle position P).

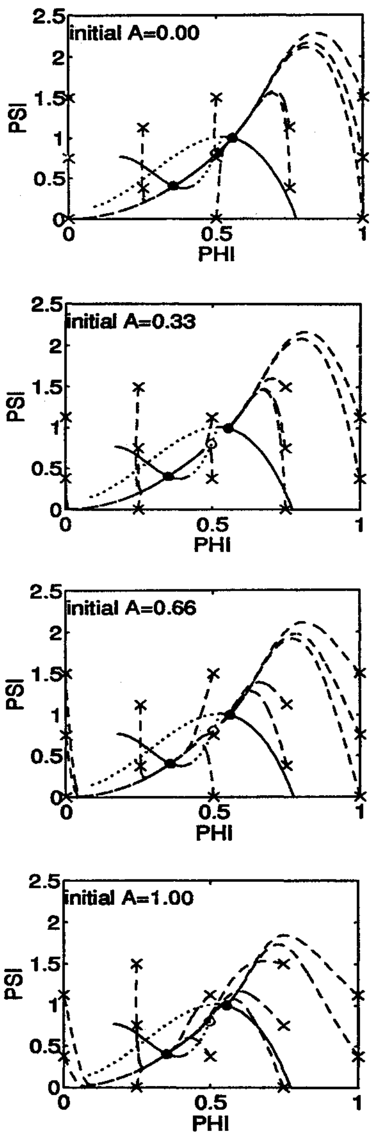

Fig. 6 Uncontrolled system trajectories (throttle position N).

those for an experimental rig discussed in Ref. 30. A model for the specific experimental rig discussed in the companion article[28] was not used in this article, because from a quantitative standpoint an accurate model of this rig has not yet been obtained. This is partially due to certain behaviors that occur in this rig that have not yet been modeled in a quantitatively accurate manner. These behaviors are discussed further in Part 2 and include the existence, when rotating stall is not present, of two first-mode rotating stall precursors rotating at different speeds in the compressor. One final point that must be made about the model used to illustrate key concept and points in this article involves including the effects of persistent disturbances in this model. The effects of all persistent disturbances could not be included in the model. Thus, a generic persistent disturbance mechanism, consisting of a disturbance throttle valve, is included in this model, which will be used to demonstrate the effects of persistent disturbances on the system. The effect of generic persistent disturbances on the system can be determined by mapping the effect of the generic persistent disturbance into an equivalent variation in this disturbance throttle valve. To allow persistent disturbances that throttle the system towards rotating stall to be introduced by closing this disturbance throttle valve, it is partially open for stable axisymmetric operation of the compressor in the uncontrolled system. This defines a nominal position of the disturbance throttle valve relative to the view of stable axisymmetric operation of the compressor. Hence, persistent disturbances are introduced to the system by opening or closing this throttle valve from the nominal position.

II. Nonlinear Perspective of Rotating Stall

In this section, the nonlinear perspective of rotating stall is presented using a numerical bifurcation technique.[31–33] This technique enhances a global picture of the influence of system parameters on the performance of the system. The performance of axial flow compressors as the system is throttled by persistent disturbances can be explained using Fig. 1. This figure gives static bifurcation diagrams (loci of equilibria) for a compressor. These diagrams represent equilibrium values of the system as a function of the disturbance throttle position. The following plots are included: 1) the amplitude A of the circumferential perturbation in axial flow coefficient as a function of the disturbance throttle position K_D, 2) the annulus-averaged compressor flow Φ as a function of K_D, 3) the an-

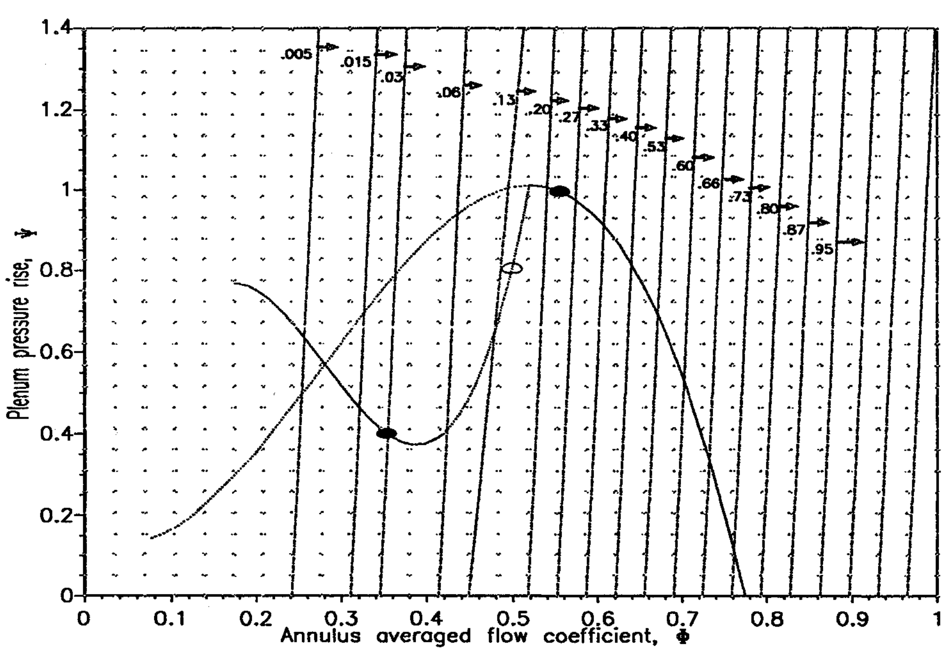

Fig. 7 Uncontrolled system—domain of attraction (throttle position N).

nulus-averaged compressor pressure rise Ψ as a function of K_D, and 4) the compressor performance characteristic of Ψ vs Φ obtained from 2 and 3 by eliminating K_D. In these plots, solid lines indicate linearly stable equilibrium points and dashed lines indicate unstable equilibrium points. The main throttle position is set to $K_C = 0.2$ in this figure and all remaining figures corresponding to the uncontrolled system. As can be seen, for high values of the disturbance throttle position, there is a single stable equilibrium point for which $A = 0$. Equilibrium points for which $A = 0$ are called axisymmetric equilibrium points. As K_D is reduced (the system is throttled up by a disturbance), a region is entered in which the axisymmetric equilibrium is no longer the only stable equilibrium. This can be seen best in the plot of A. In this region [$K_D \in (0.316, 0.461)$], there is a stable axisymmetric equilibrium $A = 0$ and a stable nonaxisymmetric equilibrium with very large amplitude $A \neq 0$. This latter equilibrium corresponds to rotating stall. The region of disturbance throttle positions in which multiple equilibria exist represents the hysteresis region of the compressor. In this region, the axisymmetric and nonaxisymmetric equilibria are not globally stable (i.e., they have finite domains of attraction). As K_D is reduced further, the axisymmetric equilibrium becomes unstable, and the nonaxisymmetric equilibrium corresponding to rotating stall is the only stable equilibrium. The point at which the stability of the axisymmetric equilibrium changes is called the theoretical (or hypothetical) stall inception point, and it is a static bifurcation point.[34] The abrupt change in the steady-state amplitude of the rotating stall at the stall inception point creates an abrupt change in the steady-state operating point on the compressor performance characteristic. (Note that the abrupt change discussed here refers to the change in steady-state rotating stall amplitude with a change in the disturbance throttle position. It does not describe the dynamics associated with the transition to rotating stall, as only steady-state information is depicted in these diagrams.)

As pointed out in Sec. I, stability of an equilibrium point deals with how a system responds when the system state is perturbed from that equilibrium point by an impulsive disturbance. The effect of impulsive disturbances on compression system operation is now discussed. In Fig. 2, the nominal disturbance throttle is set to correspond to operation at the stall inception point, which is an unstable equilibrium point. The unstable equilibrium point is indicated by the open circle in the phase-plane plot, and the stable equilibrium point is indicated by the filled circle. A perturbation on the amplitude A is introduced to account for internal system noise normally present in the compression system. The transient shows that the system moves away from the stall inception point and towards the stable nonaxisymmetric equilibrium point corresponding to rotating stall. In Fig. 3, the nominal disturbance throttle position is set at a value slightly higher than that corresponding to the hypothetical stall inception point, within the hysteresis region. A small amplitude perturbation on A is introduced to the system. This perturbation is small enough that the system returns to the stable axisymmetric equilibrium point ($A = 0$). However, if a large amplitude perturbation on A is introduced to the system, it will enter rotating stall, as shown in Fig. 4. The behavior shown in these figures illustrates the fact that the axisymmetric and nonaxisymmetric equilibrium points in the hysteresis region have finite domains of attraction. Thus, if the system state is perturbed far enough away from the equilibrium point (outside of the domain of attraction), it will not return to the equilibrium point, even though the point is linearly stable.

The hysteresis loop of the compressor can be explained further using Fig. 1; once again, the plot of amplitude vs disturbance throttle position is particularly useful. In this discussion, the compressor is assumed to be free of impulsive disturbances, except for those that are arbitrarily small. These arbitrarily small impulsive disturbances can cause the system to move away from an unstable equilibrium point, but they cannot perturb the system state beyond the domain of attraction of a linearly stable equilibrium point. Once the behavior of the system is understood for this case, the effects of finite impulsive disturbances (not necessarily arbitrarily small) on this behavior can be determined. The system starts with K_D set to a very high value where there can only be stable axisymmetric operation. The disturbance throttle position is then reduced and the system remains in stable axisymmetric operation until the stall inception point is reached. Once the stall inception point is reached, the system enters a large amplitude rotating stall, due to the presence of an arbitrarily small impulsive disturbance. If K_D is now increased, the system remains in rotating stall until a value of K_D is reached at which there is only a single stable axisymmetric equilibrium point. At this point the system returns to stable axisymmetric operation. However, the value of K_D at

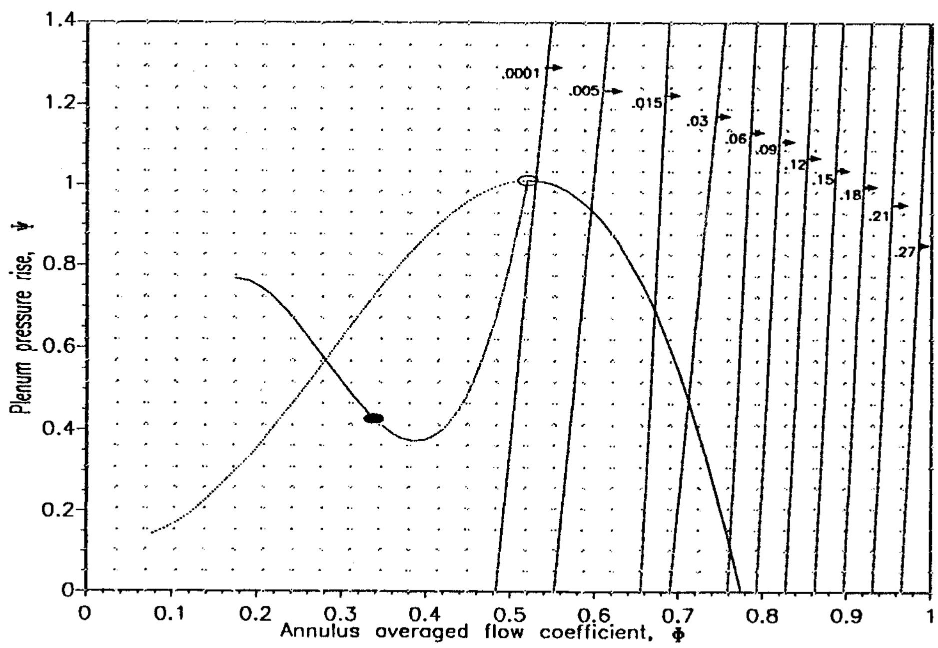

Fig. 8 Uncontrolled system—domain of attraction (throttle position P).

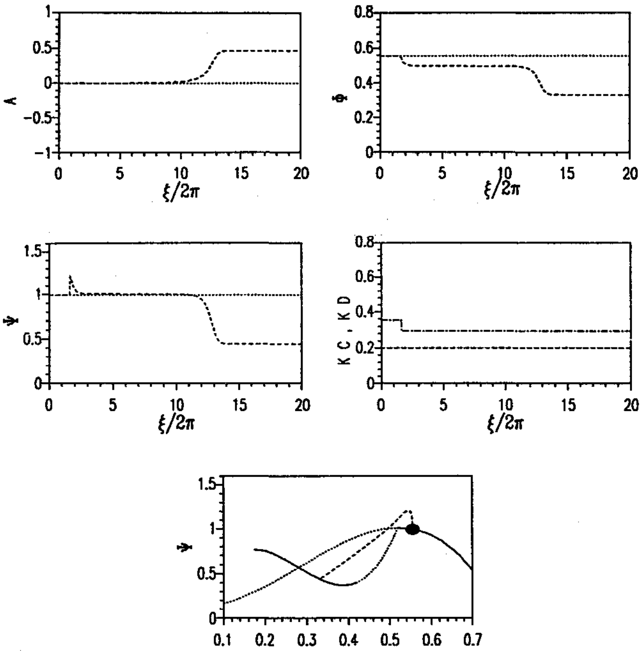

Fig. 9 Uncontrolled system response to persistent disturbance.

which this happens is much higher than the value at which the rotating stall first appeared, demonstrating the hysteresis associated with the onset and cessation of rotating stall. In other words, when the disturbance throttle position is set in this hysteresis region, the stable equilibrium state of the system can be either rotating stall or axisymmetric operation, depending on the past history of the system.

In a real compression system where finite impulsive disturbances are present, the effect of these impulsive disturbances on the point of stall inception can be quite significant. If a linearly stable axisymmetric equilibrium point has a very small domain of attraction, as is the case near the theoretical stall inception point, internal system noise can perturb the system state beyond the domain of attraction of the equilibrium point and cause the system to enter rotating stall. From a practical point of view, such equilibrium points are effectively unstable in the face of noise. For this reason, real compression systems typically enter rotating stall before the disturbance throttle position is reduced to the value corresponding to the hypothetical stall inception point. Furthermore, if a control approach enlarges the domains of attraction of such equilibrium points, it is possible that the effective stable flow range of the compressor could be increased, without the theoretical stable flow range being increased. Both of these points are demonstrated in Part 2 of this article.[28]

The concept of domains of attraction of an equilibrium point can be further illustrated using Figs. 5 and 6, which show transients plotted in the flow-pressure phase plane for various initial conditions at two different disturbance throttle positions. The filled circles denote stable equilibrium points of the system, open circles denote unstable equilibrium points of the system, and the Xs denote initial conditions that would be established by various impulsive disturbances. When the nominal disturbance throttle position is set to correspond to operation at the hypothetical stall inception point (point P), Fig. 5 suggests that, unless the initial A is zero, the system would settle into rotating stall from any point in the phase space. In a real compression system, internal noise forces A to be nonzero; hence, the plots for which the initial A is not zero show what is most likely to occur in the real system at this throttle setting. Hence, in the presence of noise, an initial condition at any point in the phase space will result in the system moving to the nonaxisymmetric equilibrium for this

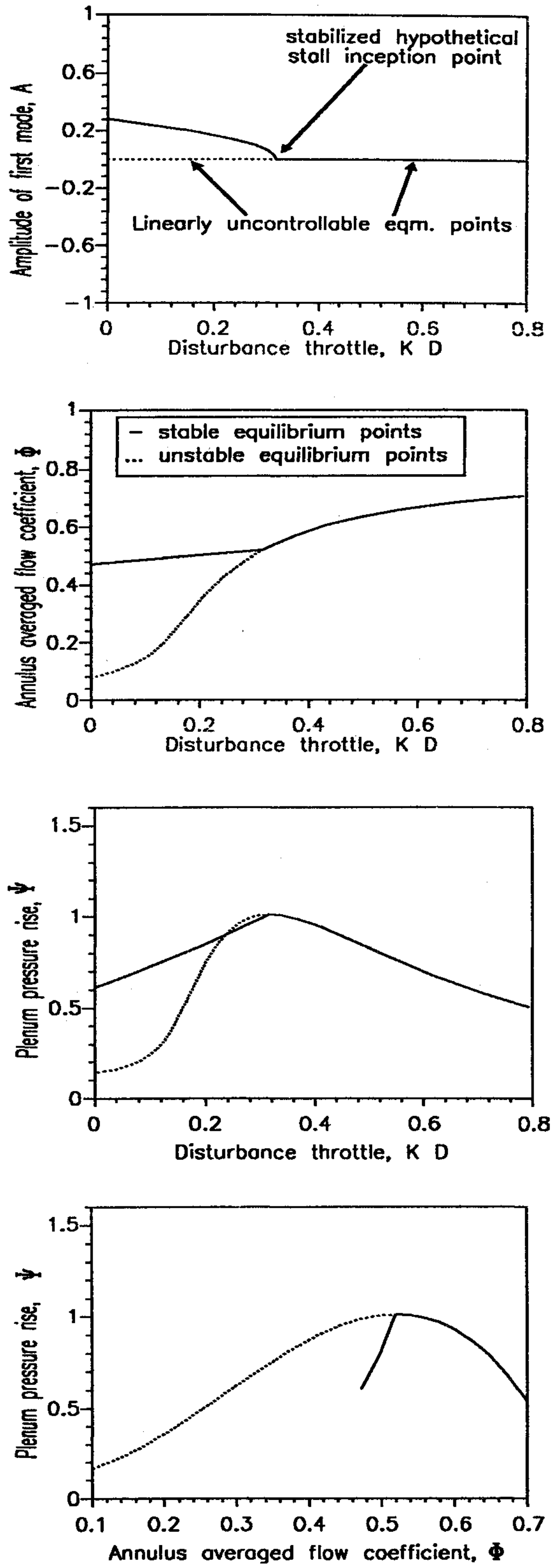

Fig. 10 Controlled system bifurcation diagram.

throttle position. Figure 6 demonstrates clearly the fact that equilibrium points within the hysteresis region possess finite domains of attraction. In this case, the nominal disturbance throttle position is set to a value slightly higher than that corresponding to the hypothetical stall inception point (point N). At this nominal disturbance throttle setting Fig. 6 suggests that, unless the initial A equals zero, the system can go to either the axisymmetric equilibrium or the nonaxisymmetric equilibrium. When the initial A equals 0, all trajectories go to the axisymmetric equilibrium point. However, this behavior would not be present in the real compression system due to internal noise. In the presence of noise, the system may or may not enter rotating stall, depending on the level of noise. Both linearly stable equilibrium points in Fig. 6 have finite domains of attraction. Since the rotating-stall equilibrium has a finite domain of attraction in the hysteresis region, it is possible to get out of rotating stall without changing the disturbance throttle position. This requires that an impulsive disturbance be introduced to the system that perturbs the system outside the domain of attraction of this equilibrium point. This suggests a possible basis for a low control authority stall recovery scheme. Though not investigated here, the authors plan to investigate such a strategy in future work.

The effectiveness of a controller in stabilizing an equilibrium point can be evaluated from the increase in domains of attraction of the controlled equilibrium point, as compared to the uncontrolled equilibrium point. The principal approaches to estimating the domains of attraction can be broadly categorized as either a Lyapunov-based approach or a direct computation approach. The Lyapunov-based methods rely on maximizing a scalar energy function over a region in phase space for which its gradient with respect to time is negative definite.[35] In higher dimensional systems, these estimates are often quite conservative. The direct computation approach, though more computationally intensive, yields exact boundaries of domains of attraction of each equilibrium point.

In this article, a direct computation approach was utilized[24,36] to evaluate the boundaries of the domains of attraction of the axisymmetric equilibrium points in Figs. 5 and 6. This approach involves forward-time integration of the system trajectories from selected initial conditions. We start by focusing on a finite, but sufficiently large, region of the phase space, which is defined by a box that encloses the axisymmetric equilibrium point. Next, a uniform grid of points is set up in this region of the phase space, and using each grid node point as the initial condition, the model equations are integrated up to time $t^* >> 0$, which is sufficiently large to be considered $t = \infty$. The proximity of the trajectory at this time to the axisymmetric equilibrium point is then determined. The boundaries of the set of initial conditions in the phase space that converge to the equilibrium point of interest are traced out as contour lines, one line for each value of initial A. In Fig. 7, these contour lines are shown in the two-dimensional phase-plane plot for the system with nominal disturbance throttle position set to correspond to operation in the hysteresis region (point N). The uniform computational grid is shown on the background as dots. The directional arrows indicate what side of the boundaries contains points that are interior to the domain of attraction of the axisymmetric equilibrium point. The $A = 0$ contour line is not shown because for this case all trajectories converge to the axisymmetric equilibrium point. Figure 7 indicates that for sufficiently small initial values of A there is a neighborhood about the axisymmetric equilibrium point in the phase plane for which all points of the neighborhood are in the domain of attraction. Thus, the axisymmetric equilibrium point is locally stable. Figure 8 shows the contour lines in the two-dimensional phase-plane plot for the system with the disturbance throttle position set to correspond to operation at the hypothetical stall inception point (point P). It is seen here that in the limit as amplitude tends to zero, the contour lines asymptotically approach the stall inception point in the phase plane. This suggests that the stall inception point is locally unstable, consistent with the behavior shown in Fig. 5. (This fact can be proven by applying the center manifold theorem[24] to the model used to generate these simulations.)

In Fig. 9, the effect of persistent disturbances on the uncontrolled system is illustrated. The system starts in stable axisymmetric operation with the nominal K_D set to a value corresponding to operation within the hysteresis region. An initial value of $A = 1 \times 10^{-4}$ is utilized to introduce an arbitrarily small impulsive disturbance to the system. A step change is made in the disturbance throttle position to create a persistent disturbance in the system. The persistent disturbance throttles the system equilibrium into the unstable axisymmetric flow range of the compressor. Since the new axisymmetric equilibrium point is unstable, the system cannot remain in axisymmetric operation in the face of arbitrarily small impulsive disturbances, and the system enters rotating stall. Thus, persistent disturbances that throttle the system equilibrium into the unstable axisymmetric flow range of the compressor in the face of arbitrarily small impulsive disturbances will cause the system to enter rotating stall.

III. Active Control

When implementing active stall control, it is important to distinguish between 1) setting/maintaining a desired equilib-

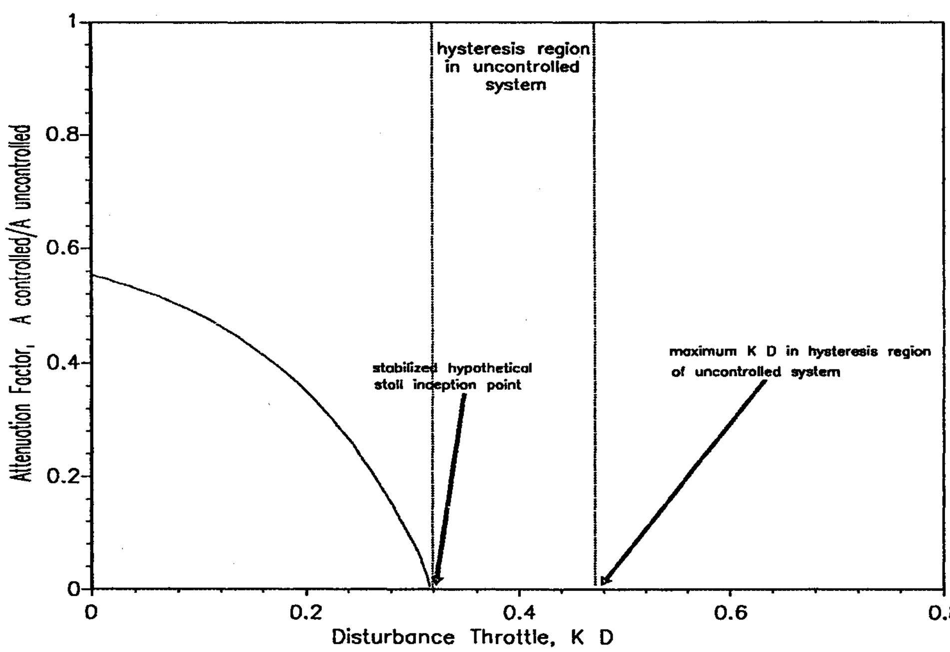

Fig. 11 Disturbance rejection in controlled system.

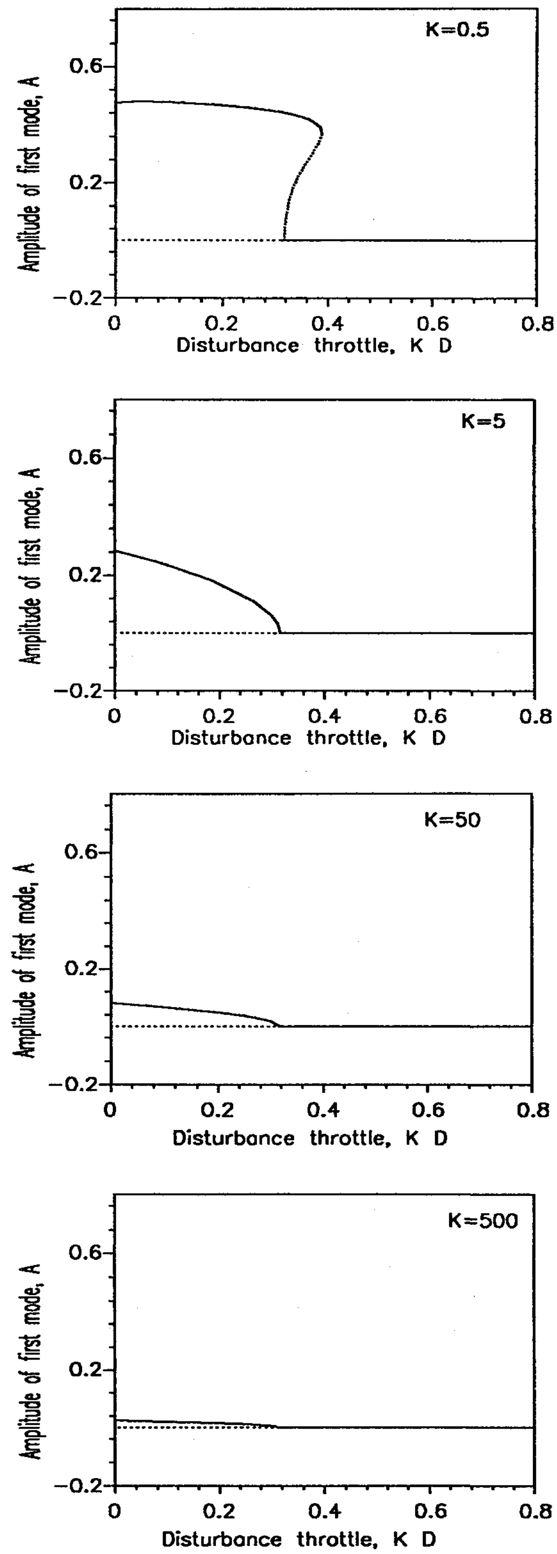

Fig. 12 Controlled system amplitude bifurcation diagrams for different controller gains.

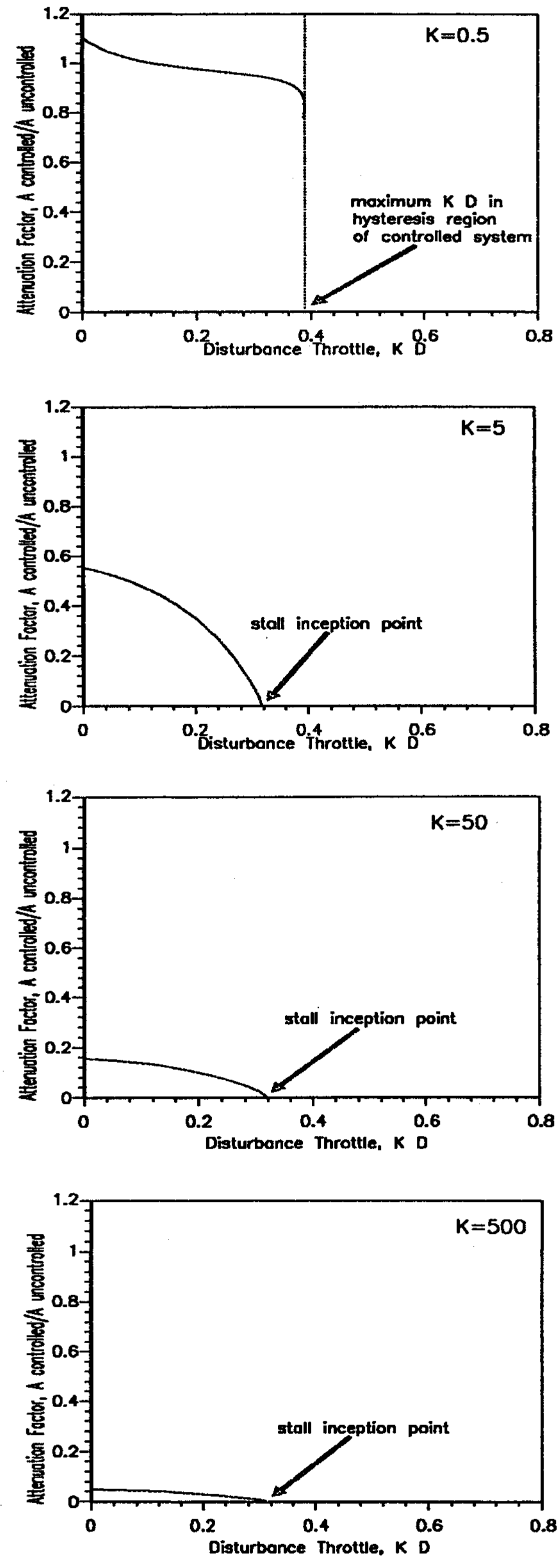

Fig. 13 Disturbance rejection in controlled system for different controller gains.

rium point in the face of persistent disturbances and 2) achieving a sufficient domain of attraction of a desired equilibrium in the face of impulsive disturbances. These represent the control requirements of 1) disturbance rejection and 2) stability augmentation, respectively. As discussed in Sec. I, the simplified approach to control of rotating stall utilizes this viewpoint to achieve control of rotating stall.

In Fig. 1, it is easy to see that if the domains of attraction of the stable axisymmetric equilibrium points in the hysteresis region are made infinite, the stable nonaxisymmetric equilibrium branch could only be present for values of disturbance throttle position less than that corresponding to the stall inception point. In this case, the static bifurcation diagrams of such a controlled system would be of a form similar to that shown in Fig. 10. In this figure, the stable axisymmetric equilibrium is the only possible equilibrium in the controlled system for all values of K_D greater than that corresponding to the stall inception point. For values of nominal throttle less than that corresponding to the stall inception point, the axisymmetric equilibrium is still unstable. The abrupt jump in steady-state rotating stall amplitude when the system enters rotating stall has been eliminated. Additionally, hysteresis with respect to the onset and cessation of rotating stall has been eliminated. Thus, no abrupt change occurs in the steady-state compressor operating point when the controlled system does enter rotating stall. When rotating stall does initially occur in the controlled system, the system operates in a small amplitude rotating stall that results in only a small change in the steady-state compressor operating point. Moreover, since the locally stable axisymmetric equilibrium points are now globally stable, the possibility for internal noise to perturb the system state beyond the domains of attraction of these points is eliminated, and extension of the effective stable flow range is achieved. The disturbance rejection achieved by such a controller is clearly evident by comparing the values of A for the same K_D in both the controlled and uncontrolled system. In Fig. 11, a plot of an attenuation factor $A_{\mathrm{controlled}}/A_{\mathrm{uncontrolled}}$ as a function of K_D is given. For purposes of this plot, the ratio of 0/0 when it occurs is set equal to 0. This plot indicates that the rotating stall amplitude is either significantly reduced (for K_D values less than that corresponding to the stall inception point) or completely eliminated (for K_D values corresponding to the hysteresis region of the uncontrolled system) by such a controller.

As mentioned earlier, it is expected that this control approach will involve significantly reduced controller complexity in terms of actuation requirements. In fact, results have been obtained that indicate that this approach can be implemented with two-dimensional sensing and only a single one-dimensional axisymmetric effector. In Ref. 19, one possible control strategy was given that could be used as part of the simplified approach for control of rotating stall. This strategy involves feeding back the square of the amplitude A to the main throttle according to the control law $K_C = K_{C_{\mathrm{nom}}} + kA^2$. This control strategy was demonstrated using the first-term Galerkin approximation of the Moore–Greitzer model. It was shown[19] that the uncontrolled bifurcation diagram of this model could be modified to the form shown in Fig. 10 using this control strategy (in fact, this controller structure with $k = 5.0$ and $K_{C_{\mathrm{nom}}} = 0.2$ was used to generate Fig. 10). In Fig. 12, the effect of the controller gain k on the bifurcation diagram for A vs K_D is shown. As the gain is increased, the magnitude of the steady-state rotating stall amplitude for a given value of K_D is reduced. Thus, the level of disturbance

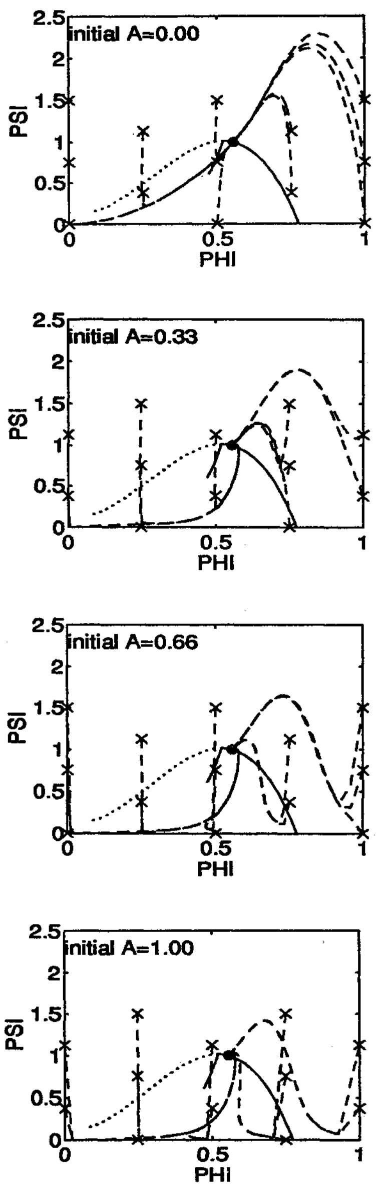

Fig. 14 Controlled system trajectories (throttle position N).

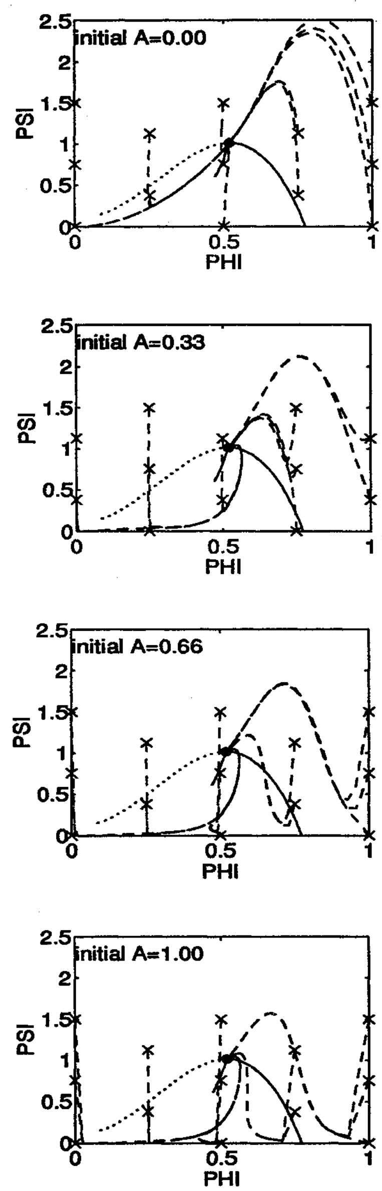

Fig. 15 Controlled system trajectories (throttle position P).

rejection achieved by the controller is increased as the gain is increased. This can be illustrated by plotting the attenuation factor as a function of K_D for different values of gain, as shown in Fig. 13. As can be seen, the attenuation factor goes down as the gain is increased. The magnitude of the perturbation in steady-state compressor operating point (flow and pressure rise) is also further attenuated as the gain is increased. One potential drawback of this particular control strategy, as implemented in Ref. 33, lies in the requirement of flow measurement, which is inherently noisy due to the intrusive nature of flow sensors,[37] to obtain A. However, in Part 2 of this article,[28] it is shown that this type of controller can be implemented using six static pressure sensors that are significantly less noisy.

The concept of increased accommodation of impulsive disturbances by enlarging domains of attraction in the controlled system are fully demonstrated by the trajectories shown in the phase-plane plots given in Figs. 14 and 15 for nominal K_D values corresponding to operation at points N and P, respectively. The controller gain k is set to 5.0 and $K_{C\text{nom}} = 0.2$ in these figures and all remaining figures pertaining to this controller. The same initial conditions (established by impulsive disturbances) as those in Figs. 6 and 5 are used in Figs. 14 and 15, respectively. Figure 14 illustrates the fact that in the controlled system at this nominal disturbance throttle setting (point N), the axisymmetric equilibrium is now a global attractor. This point will be approached asymptotically in time for any initial condition, including those far from the steady-state locus (i.e., large perturbations in amplitude, flow, or pressure). Figure 15 illustrates that the theoretical stall inception point can be made a global attractor with this controller using a one-dimensional axisymmetric effector. This is not inconsistent with the linear analysis since there is an eigenvalue on the $j\omega$ axis for this point and the local dynamics about this point cannot be determined from the linear analysis.[19] Thus, linear stabilization of the stall inception point with one-dimensional axisymmetric actuation is not precluded, as is the case for all other axisymmetric equilibria based on the linear analysis.

Since this controller does not require or act upon rotating stall phase information, the bandwidth requirements of this controller are relatively low. To demonstrate that this control

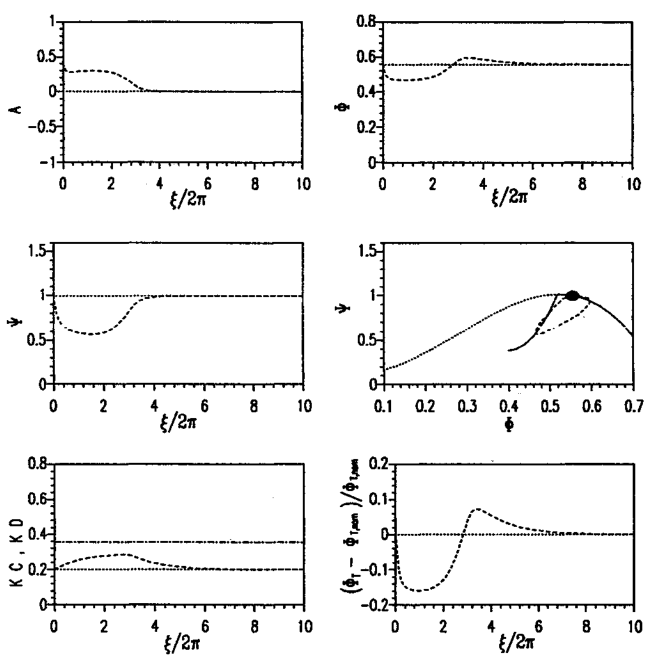

Fig. 16 Controlled system response to large non-equilibrium initial condition on A in former hysteresis region with actuator lag and constraint.

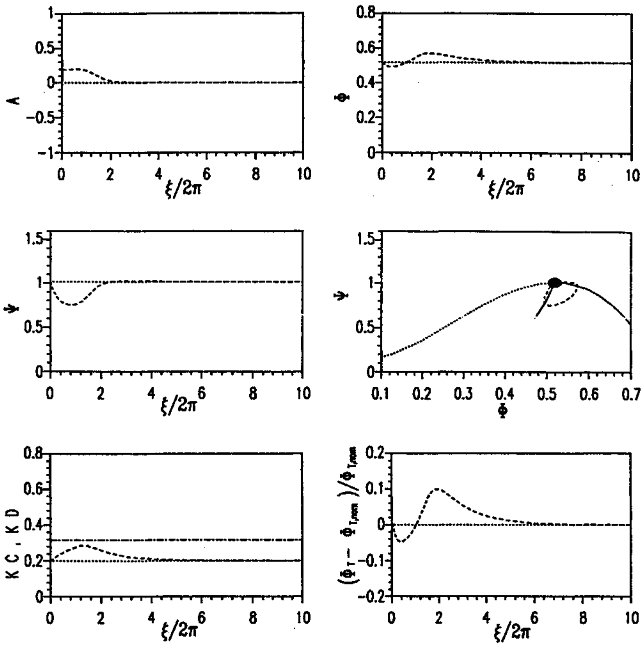

Fig. 17 Controlled system response to nonequilibrium initial condition on A at stall inception point with actuator lag and constraint.

strategy can be effected in the face of bandwidth constraints, a first-order lag was implemented on the actuator of the form

$$\dot{K}_C = (1/\tau)(K_{C\text{dem}} - K_C)$$

where $K_{C\text{dem}} = K_{C\text{nom}} + kA^2$. In addition, a constraint was placed on $K_{C\text{dem}}$ to demonstrate that this controller does not require excessive control action to be effective. The constraint was placed on the demanded value of throttle position, as this is what would be implemented in a physical system so that the actuator would not be asked to exceed its physical limitations by the controller. The cutoff frequency of this actuator lag is approximately the rotational frequency of the compressor. This should not be taken as a concrete indication of the required actuator bandwidth for success of this type of controller. In fact, in the companion article,[28] it will be shown that with actuator bandwidths, much less than the rotational frequency of the compressor, success of this type of controller can be achieved. With an actuator that has 70-Hz bandwidth and a compressor with a rotational frequency of 180 Hz, similar success is achieved with this type of controller. The maximum $K_{C\text{dem}}$ is set to a value that is small enough so that the controller cannot simply throttle the system beyond the steady-state hysteresis region of the uncontrolled system to eliminate rotating stall. In Fig. 16, the same scenario as that in Fig. 4 is shown for the controlled system with the actuator lag and constraint included in the controller. Recall that in this scenario the compressor is operating in the hysteresis region of the uncontrolled system (point N) and the initial condition on amplitude A is large enough that the system enters rotating stall when uncontrolled. As can be seen in Fig. 16, even with the constraint and the actuator lag, the controlled system returns to axisymmetric operation. In addition to previously shown plots, Fig. 16 shows a plot for the control effort that is computed as

$$\frac{\Phi_T - \Phi_{T\text{nom}}}{\Phi_{T\text{nom}}}$$

where Φ_T and $\Phi_{T\text{nom}}$ are the actual and nominal overall throttle flows, respectively. As is indicated by this plot, the control action shows a maximum change of 25% in nominal throttle flow in this scenario. In Fig. 17, the nominal throttle position

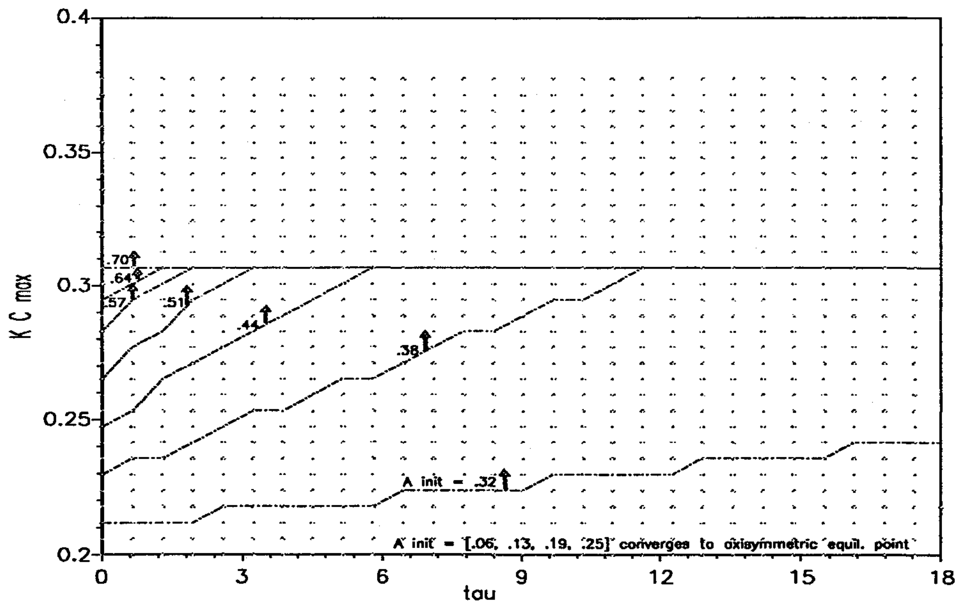

Fig. 18 Controlled system—effect of actuator lag and constraint (throttle position N).

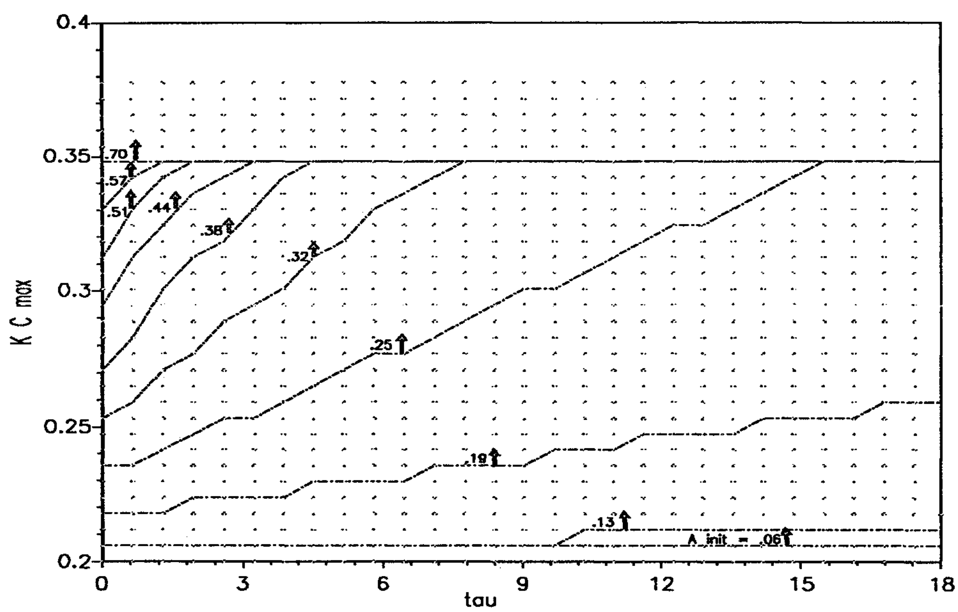

Fig. 19 Controlled system—effect of actuator lag and constraint (throttle position P).

is set to a value that corresponds to operation at the stall inception point (point P), and the same actuator lag and constraint are again included in the controller. The system returns to axisymmetric operation from an initial condition on A, which is the same as that in Fig. 2 for which the uncontrolled system entered rotating stall. As is indicated in this figure, the control action shows a maximum change of 15% in nominal throttle flow.

Figures 18 and 19, respectively, show the effect of the actuator lag and constraint on the domains of attraction of the axisymmetric equilibria in the controlled system at the nominal operating points N and P. In these plots, dash–dot lines for different values of initial amplitude A are shown. Each of these lines divides the plane of the actuator lag and constraint into one region for which the value of initial A is in the domain of attraction of the equilibrium point and one for which it is not. The arrows point to the region for which the value of A is in the domain of attraction. Initial values of Ψ and Φ are set to those for the axisymmetric equilibrium. In Fig. 18, for the lowest values of initial A considered, all trajectories return to the axisymmetric equilibrium point for the values of actuator lag and constraint considered. Several conclusions can be made using Figs. 18 and 19, along with the trajectory plots given in Figs. 14 and 15 for which no actuator lag or constraint are included in the controller. First, with no actuator lag or constraint, the domain of attraction of both axisymmetric equilibrium points appears to be infinite. Second, if the constraint is high enough such that the controller can throttle the system beyond the steady-state hysteresis region of the uncontrolled system, all values of initial A will result in the system returning to the axisymmetric equilibrium regardless of the actuator lag. Third, if the constraint is low enough that the controller cannot throttle the system beyond the steady-state hysteresis region of the uncontrolled system, there is a maximum value of initial A, which can be tolerated with the system still returning to the axisymmetric equilibrium. For this case, as the lag is increased, the maximum value of initial A that can be tolerated is decreased. This allows the possibility of the system to enter rotating stall without the controller having enough actuator travel to return the system to the axisymmetric equilibrium. However, even with actuator lags and constraints, the controlled system can tolerate a much higher level of impulsive disturbances in A than in the uncontrolled system, while still returning to axisymmetric operation. In fact, the level of impulsive disturbances in A that can be tolerated in many cases is on the same order as that for a fully developed rotating stall. Thus, attenuation of the effects of such disturbances has been achieved with this controller, even when actuator lags and constraints are included in the controller.

In Fig. 20, the effect of introducing a persistent disturbance to the controlled system is illustrated. The same persistent disturbance that was introduced to the uncontrolled system in Fig. 9 is introduced to the controlled system. Recall that this persistent disturbance caused the uncontrolled system to enter a large-amplitude rotating stall, in the face of an arbitrarily small impulsive disturbance. The same arbitrarily small impulsive disturbance is also utilized here. The same actuator lag and constraint as in the scenario of Fig. 16 are utilized. As can be seen in Fig. 20, the controller attenuates the effect of this persistent disturbance, and the system enters a very low amplitude rotating stall. Also indicated by this figure is that the control action is not excessive in this scenario, despite the imposed constraint.

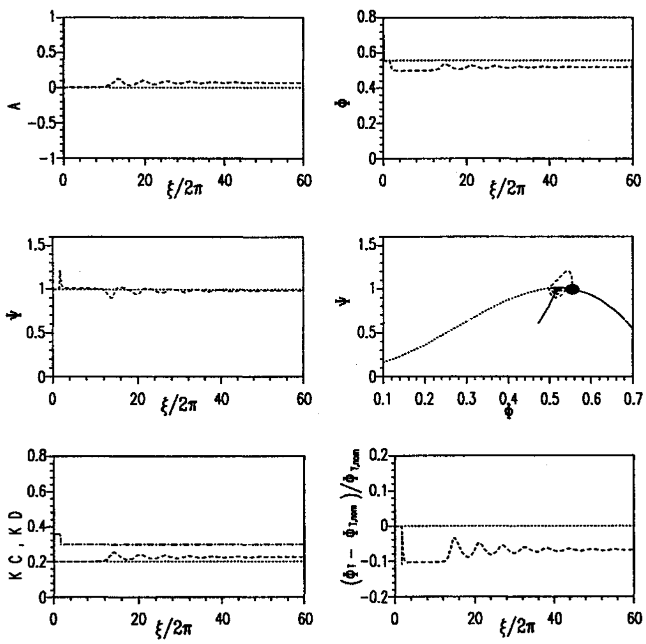

Fig. 20 **Controlled system response to persistent disturbance with actuator lag and constraint.**

The notion of using feedback disturbance rejection to attenuate the effects of disturbances that result in compressor stall phenomena was discussed previously.[12] Also, the notion of feedback of rotating stall amplitude to the throttle has been discussed.[31,33,38] There, however, the control strategy was for rotating stall avoidance and not control as described here. This can be seen in Fig. 8 of Ref. 38 in which the nominal operating point has been moved by the controller to a point in the stable region of compressor operation.

It is suspected that the experimentally implemented rotating stall controller presented in Ref. 13 also achieves some of the objectives of the simplified approach for control of rotating stall presented here. With this controller, rotating stall control was achieved with one-dimensional axisymmetric actuation in the form of air jets blowing synchronously through an array of equally spaced slots around the circumference of the compressor. This actuator was controlled in accordance with a measurement of the flow asymmetry. It was shown that limited extension (6%) of the stable flow range could be obtained with this type of control. This could perhaps be attributed to an extension of the effective stable flow range that was obtained by enlarging the domains of attraction of linearly stable equilibrium points. However, the experimental results presented in Ref. 13 do not allow a definite conclusion to be made in this regard, and further experimental investigation of this controller would be required to bear out this conjecture.

In Part 2 (Ref. 28) of this article a control scheme based on the approach of this article is implemented in a single-stage, low-speed axial compressor rig to demonstrate exper-

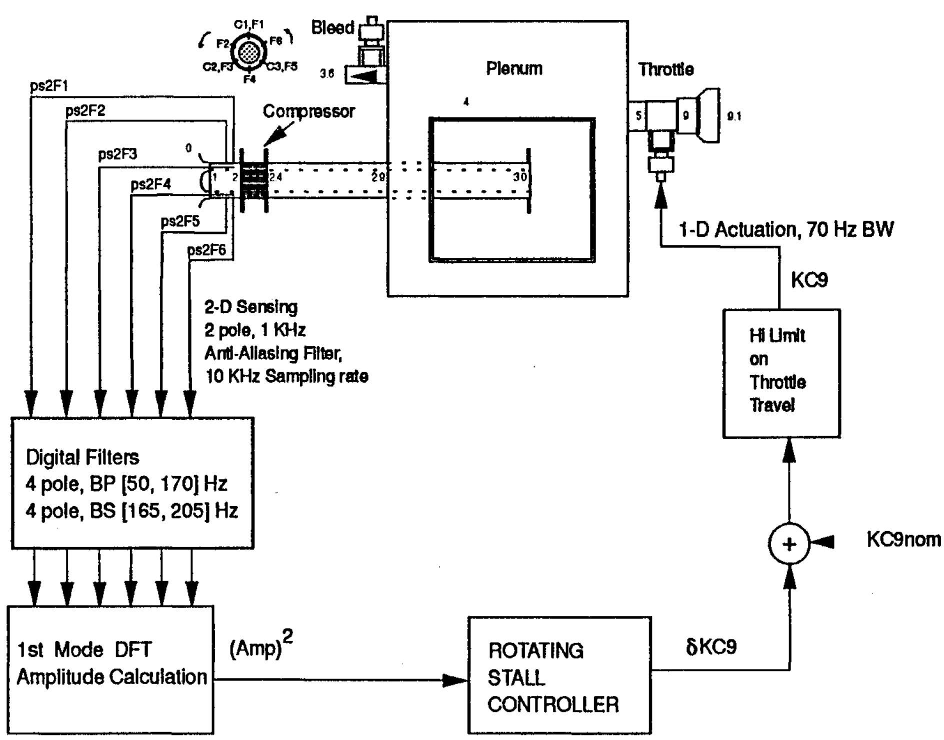

Fig. 21 Rotating stall controller diagram.

imentally the merits of this simplified approach for control of rotating stall. A diagram of the controller implemented in experiment is given in Fig. 21. In this controller, six static pressure sensors at the inlet of the compressor are used after appropriate filtering to compute the magnitude of the first discrete Fourier transform (DFT) coefficient of the traveling wave present in rotating stall. The square of this magnitude multiplied by a gain is fed back to the throttle. The success of this control scheme is demonstrated experimentally in Part 2 of this article (Ref. 28). In addition, qualitative agreement is shown between the nonlinear behaviors discussed in this article and those observed in experiment for both the uncontrolled and controlled systems.

IV. Conclusions

In this article, the theoretical foundations of a simplified approach for control of rotating stall were developed. This approach requires two-dimensional sensing, but only a single one-dimensional axisymmetric effector with relatively low bandwidth requirements. The reduced actuation requirements of this approach were a consequence of the fact that in this approach one does not require or act upon rotating stall phase information. This is due to the fact that one does not seek to extend the theoretical stable axisymmetric flow range of the compressor. Rather, the controller directly addresses persistent disturbances that would otherwise throttle the equilibrium into the unstable axisymmetric flow range of the compressor. In addition, one seeks to enlarge the domains of attraction of linearly stable axisymmetric equilibria, thereby addressing impulsive disturbances that would otherwise perturb the system state beyond the domain of attraction of the stable axisymmetric equilibrium. By using this approach, one eliminates 1) the abrupt jump in steady-state rotating stall amplitude when the system enters rotating stall and 2) hysteresis with respect to the onset and cessation of rotating stall. Thus, no abrupt change occurs in the steady-state compressor operating point when the controlled system does enter rotating stall. Experimental validation of this approach on a single-stage low-speed axial compressor rig is discussed in Part 2 of this article.

Extensions for the concepts presented in this article that should be considered include further investigation to address more directly: 1) transients, 2) inlet distortion, and 3) compression system uncertainties. The command following capability of feedback can be utilized to attenuate the effect of transients, thereby reducing operating line excursions during acceleration and deceleration transients. The concept of feed-forward disturbance rejection can be used to attenuate the effects of inlet distortion, since this disturbance is measurable, and information on this disturbance can be fed forward in the controller. The notion of using feed-forward disturbance rejection to attenuate the effects of compression system disturbances that result in compressor stall phenomena was discussed previously.[12] As mentioned in Sec. I, the effects of compression system uncertainties can be addressed through the robustification of controllers by the application of well-established robust control design methodologies.[23] Finally, additional stability augmentation in the compression system may be achievable using the concept of vibrational control,[39] which involves an open loop time-varying controller that requires no sensing.

Appendix: Moore–Greitzer Model

In this section, the first-term Galerkin approximation of the rotating stall model developed in Ref. 29 is reviewed. This rotating stall model often referred to as the Moore–Greitzer model, involves PDEs with integral boundary conditions, and hence, exact solutions to this model may be computationally intensive. The first-term Galerkin approximation of the Moore–Greitzer model is comprised of a set of three first-order integro-differential equations. The three state variables are 1) the dimensionless amplitude of the first-term circumferential disturbance in the axial flow coefficient in the compressor $A(\xi)$, 2) the dimensionless annulus-averaged axial flow coefficient $\Phi(\xi)$, and 3) the dimensionless annulus-averaged plenum pressure rise $\Psi(\xi)$, where ξ is the dimensionless time variable. In this model, the local total-to-static pressure rise across the compressor is defined in terms of an axisymmetric characteristic $\psi(\phi)$, which is a function of the local axial flow coefficient $\phi = \Phi + A \sin \zeta$. Here, ζ is the angular position in the rotating frame of the circumferential disturbance in axial flow coefficient. The resulting model equations, expressed in terms of the dimensionless variables defined in Ref. 29, are

$$\frac{\mathrm{d}A}{\mathrm{d}\xi} = \frac{\alpha}{\pi}\int_0^{2\pi} \psi(\Phi + A \sin \zeta)\sin \zeta \, \mathrm{d}\zeta \tag{A1}$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\xi} = \frac{1}{l_c}\left[\frac{1}{2\pi}\int_0^{2\pi} \psi(\Phi + A \sin \zeta)\, \mathrm{d}\zeta - \Psi\right] \tag{A2}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\xi} = \frac{1}{4B^2 l_c}[\Phi - F_T^{-1}(\Psi)], \qquad F_T = \frac{\Phi_T^2}{K_T^2} \tag{A3}$$

Here, the dimensionless parameters l_c and B have identical definitions to that presented in Ref. 29. The parameter α is defined as $\alpha = a/(1 + ma)$, where m and a are defined in Ref. 29. In addition, the throttle parameter K_T is defined here as $K_T = \sqrt{2}(a_T/a_c)$, where a_T and a_c denote the flow areas of the throttle and the compressor, respectively. For purposes of discussion in this article, the throttle parameter is considered to be made up of two parts, $K_T = K_C + K_D$. Here, K_C is associated with a main throttle valve that will be used for control, and K_D is associated with a disturbance throttle valve that will be used to introduce persistent disturbances to the system. In solving the model equations, the integral on the right-hand side of Eqs. (1) and (2) has to be evaluated at each instant of time. These integrals are computed analytically using the following cubic axisymmetric characteristic: $\psi(\phi) = -16.0783\phi^3 + 13.5102\phi^2 - 1.01739\phi + 0.148883$. This characteristic is a best fit cubic to the compressor characteristic labeled C2 in Ref. 30. The model parameters for this system are $l_c = 6.66$, $B = 0.1$, and $\alpha = 0.3039$.

Acknowledgments

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